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Models for Flare Statistics and the Waiting-time Distribution of Solar Flare Hard X-ray Bursts

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Abstract

In a previous study (Wheatland, Sturrock, McTiernan 1998), a waiting-time distribution was constructed for solar flare hard X-ray bursts observed by the *ICE/ISEE-3* spacecraft. A comparison of the observed distribution with that of a time-dependent Poisson process indicated an overabundance of short waiting times (10 s – 10 min), implying that the hard X-ray bursts are not independent events. Models for flare statistics assume or predict that flares are independent events – in particular the avalanche model makes this specific prediction. The results of the previous study may be reconciled with the avalanche picture if individual flares produce several distinct bursts of hard X-ray emission. A detailed comparison of the avalanche model and the *ICE/ISEE-3* waiting-time distribution is presented here.

Key words: Sun: activity — Sun: corona — Sun: flares

1. Introduction

Studies of solar flare statistics may allow insight into the mechanisms of energy storage and release in flares. An important development has been the recognition that the frequency distributions of hard X-ray event size (and possibly also duration) are power laws (Hudson 1991). These observations motivated a number of models for flare statistics, including cellular automaton (CA), or avalanche models, based on the ideas of self-organised criticality (e.g. Lu, Hamilton 1991, Macpherson, MacKinnon 1997), and analytic descriptions of the energy balance in an active region (e.g. Rosner, Vaiana 1978, Litvinenko 1994). Recently the analytic approach was developed into a general formalism based on a master equation for the probability that an active region has a given free energy (Wheatland, Glukhov 1998).

One statistical property of flares that is of interest is the waiting-time distribution (WTD), or the distribution of times between events. This has received little attention by comparison with the flare size distribution (see, however, Biesecker 1994; Pearce, Rowe, Yeung 1993; Crosby 1996). The WTD provides information about whether flares are independent events, or whether the occurrence of one flare makes another more or less likely. There is a variety of reports in the literature of sympathetic flaring, i.e. the triggering of one flare by another. It is also conceivable that the occurrence of a large flare makes a subsequent flare less likely (e.g. because energy has been depleted). Both possibilities may be examined via the WTD.

Models for flare statistics make predictions or assumptions about the waiting-time distribution. For example, the Rosner and Vaiana (1978) model begins by assuming that flares occur as a Poisson process in time and so have an exponential WTD. The avalanche model of Lu and Hamilton (1991) predicts that the WTD is exponential, as may be verified by examining the results of CA calculations (e.g. see Fig. 1).

In a previous study (Wheatland, Sturrock, McTiernan 1998; hereafter WSM98), a WTD was constructed for eight years of hard X-ray bursts observed by the *ICE/ISEE-3* spacecraft. We begin by briefly summarizing the results of that study, and then present a more detailed comparison of the observed WTD with that predicted by the avalanche model.

2. The WTD for the *ICE/ISEE-3* Hard X-ray Bursts

The *ICE/ISEE-3* events were selected in a prior statistical study (Bromund, McTiernan, Kane 1995), and (as in any such study), arbitrary decisions were made in the selection procedure. Consequently, we label the events as hard X-ray bursts, rather than presupposing that each burst represents a "flare."

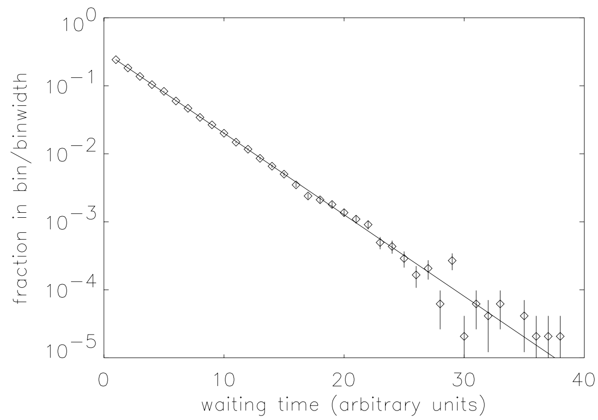

Fig. 1.. WTD constructed from a run of the Lu and Hamilton (1991) solar flare model, and a fit to an exponential.

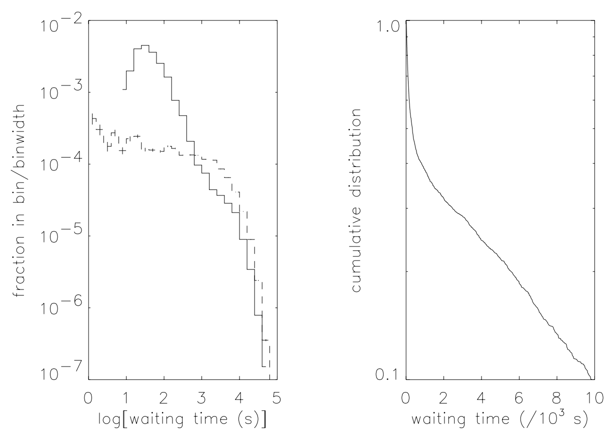

Fig. 2.. Two ways of looking at the WTD for the *ICE/ISEE-3* bursts: at left as a log-log plot of the differential distribution, and at right as log-linear plot of the cumulative distribution. The left panel also shows the result of a simulated time-dependent Poisson process with the same history of rates (dashed histogram).

Fig. 2 shows the waiting-time distribution that was constructed in WSM98 from 3574 *ICE/ISEE-3* events. In the left panel the WTD is shown in a log-log representation (the solid histogram). The distribution is not obviously a simple exponential form (cf. Biesecker 1994) or a power-law (cf. Pearce, Rowe, Yeung 1993), or a combination of the two (cf. Crosby 1996). In WSM98 the observed WTD was compared with a time-dependent Poisson process, for which the rates were estimated from the data, using a Bayesian procedure due to Scargle (1998). The result of the simulation is shown as a dashed histogram in the left panel of Fig. 1. It is clear that there is an overabundance of short waiting times (10 s – 10 min) in the observed distribution in comparison with the time series of independent events. We conclude that the *ICE/ISEE-3* events are not independent.

The interpretation of this result depends on whether the *ICE/ISEE-3* bursts represent individual flares, by which we mean episodes of energy release. If they do, then we have observed sympathetic flaring, i.e. the triggering of one flare by another. An alternate possibility is that individual flares may produce several hard X-ray bursts. In this case the bursts exhibit dependency, whilst the flares themselves may be independent events. The form of the observed WTD argues in favour of this interpretation. In the right panel of Fig. 1 the WTD is again shown, but this time as a cumulative distribution (in this case it is no longer necessary to bin the events), in a log-linear representation. It is clear that the WTD qualitatively is a simple exponential form (at least for this range of waiting times) except for

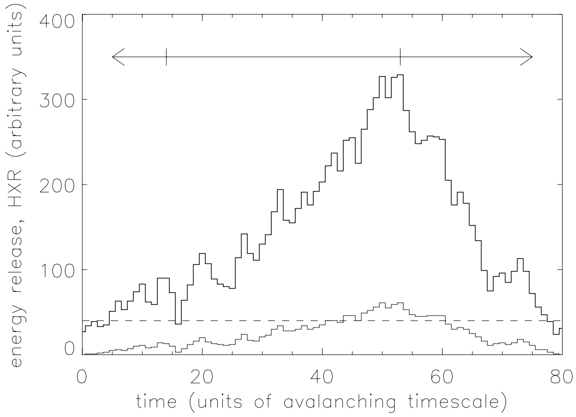

Fig. 3.. The thin histogram is a time history of unstable sites for a single avalanche in a Lu and Hamilton (1991) type CA. The thick histogram is the assumed profile of HXR. The dashed line is the threshold for event slection, and the line with ticks above indicates waiting times.

a large overabundance of short waiting times. The range of the excess corresponds well with typical flare durations, suggesting that we are looking at a departure from a Poisson process due to multiple hard X-ray bursts associated with individual flares.

3. A Further Comparison with the Avalanche Model

The avalanche model presented by Lu and Hamilton (1991) and Lu et al. (1993) produces a synthetic time history of the rate of energy release, $R(t)$. To compare the model with hard X-ray observations, there are two important considerations. First is the question of how hard X-ray production is related to energy release. If the time profile of hard X-ray production does not mimic $R(t)$, then individual flares may produce distinct hard X-ray bursts. There is evidence that sometimes flare energy release proceeds without appreciable hard X-rays (Bai, Sturrock 1989), but unfortunately we do not not understand in detail the relationship between energy release and HXR production.

Second, there is the question of how events are selected from the X-ray observations. In the following we consider whether selection effects may account for the discrepancy between the observed WTD and that predicted by the avalanche model.

We begin by making the naive assumption that hard X-ray production per timestep is linearly related to $R(t)$, with the addition of background noise:

$$R_{\mathrm{HXR}}(t) = kR(t) + \delta_{\mathrm{N}}/\Delta t_a, \tag{1}$$

where δ_N is Poisson noise with a mean of N, and Δt_a is the avalanching timestep (assumed to be unity in the following). Then, we identify events from $R_{\mathrm{HXR}}(t)$ when the counts in a timestep exceed the background by 3σ, i.e. $R_{\mathrm{HXR}}(t)\Delta t_a > 3\sqrt{N} + N$. This is similar to the selection procedure used with the *ICE/ISEE-3* events (Bromund, McTiernan, Kane, 1995) and elsewhere in flare studies (Aschwanden et al. 1998).

Fig. 3 illustrates these assumptions. The thin solid histogram shows $R(t)$ for a single avalanche in a CA of dimension 30^3 (for details of the avalanche code, see Edney, Robinson, Chisholm 1998). The thick histogram shows $R_{\mathrm{HXR}}(t)$, for $k = 5$ and $N = 25$, and the dashed line is the significance threshold. The single "flare" produces two HXR bursts, and so an additional (short) waiting time is introduced (the horizontal line with ticks above the graph shows this). There is a subtlety in the inclusion of this waiting time: in the Lu and Hamilton model there are two timescales – a slow driving between avalanches (timestep Δt_d), and avalanching, with a timestep Δt_a. We have assumed that $\Delta t_a/\Delta t_d = f$, where f is chosen so that the longest avalanches take less than one driving timestep.

The distributions resulting from the selection procedure applied to $R_{\mathrm{HXR}}(t)$ are shown in Fig. 4. It is of interest to check that the peak-flux is still a power law, since the selection procedure has introduced many smaller events. Fig. 4 confirms that this is so, although the index of the energy distribution is somewhat increased (from -2.2 to -2.6). The cumulative WTD is shown in the lower right panel, as a log-linear plot. It is clear that it qualitatively is

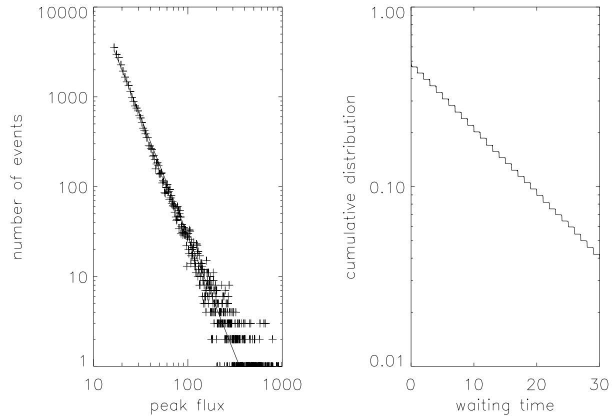

Fig. 4.. Left panel: the distribution of peak flux base on $R_{\mathrm{HXR}}(t)$ and the selection procedure. Right: the cumulative WTD is an exponential with an excess of short waiting times, reproducing the form of the *ICE/ISEE-3* distribution.

similar to the right panel in Fig. 2, consisting of an exponential (a straight line in a log-linear plot) with an excess of short waiting times (the cumulative distribution starts at one by definition, and so the plot drops steeply from one at a waiting time of zero to a value of about 0.5, and thereafter is a straight line with a flatter slope).

4. Summary

A study of the waiting-time distribution for HXR bursts observed by the *ICE/ISEE-3* spacecraft revealed an excess of short waiting times by comparison with a series of independent events with the same rates. The avalanche model for flares predicts that flares are independent events. The model may be reconciled with the observations if individual flares (energy release events) are independent, but produce multiple (dependent) HXR bursts. Two possibilities are that hard X-ray production does not mimic energy release, producing intrinsically distinct HXR bursts from a single flare. A second possibility (investigated here) is that extrinsic selection effects lead to multiple HXR events being identified from a single flare.

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